Gravitational Energy and Background Structure

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Image credit: NASA, Galileo mission (2006)

Jupiter's moon Io

Most volcanic activity in the solar system, as predicted by Peale et al. (1979) prior to Voyager, Galileo, ...

Where does the energy come from?

Tidal Heating

Net work done by an external tidal field on an isolated body (not necessarily “heat,” but usually is in astrophysical cases).

Isolated $:=$ three scales large compared to body, (i) radius of curvature, (ii) scale of inhomogeneity, and (iii) time scale for curvature changes.

Relevant for satellites; black holes; binary neutron stars; etc.

General result

$$\frac{dW}{dt} = -\frac{1}{2}\mathcal{E}_{ij}\frac{d\mathcal{I}_{ij}}{dt}, \qquad (1)$$

where W is the work, $\mathcal{I}_{ij}$ is the mass quadropole moment for the isolated body and $\mathcal{E}_{ij} := R_{i0j0}$ represents the external curvature.

(Thorne and Hartle 1985 ...Thorne 1998, Purdue 1999, Favata 2001)

Electromagnetism

Energy density of EM field (in vacuum):

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) \qquad (2)$$

Energy-momentum flux (Poynting vector):

$$S = \frac{1}{\mu_0}(E \times B) \qquad (3)$$

Conservation (in vacuo):

$$-\frac{\partial}{\partial t}\int_v u dV = \oint_{\partial V} S \cdot dA + \int_V J \cdot E dV \qquad (4)$$

(J is current density.)

Radiative Heating

Conversion of EM energy into other forms.

Conservation principle applies, can be used to calculate amount of radiative heating.

Tidal Heating

There are no local analogs of expressions for:

- Energy density of gravitational field
- Energy-momentum flux

And hence no integral form of conservation, over finite regions

How to understand the work done / energy-momentum flux in tidal heating? (... and energetic concepts in a variety of other applications?)

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  Need analog of "inertial structure," to define work as deviation from freely falling motion over some distance. Justification for doing this in specific modeling contexts.
- Problems with quasi-local accounts

Outline

1. Eliminativists
2. Going Quasi-Local
3. Challenges

Anybody who looks for a magic formula for "local gravitational energy-momentum" is looking for the right answer to the wrong question.

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It is perhaps ironic that energy conservation, a paradigmatic physical concept arising initially from [Galileo] ... should nevertheless have found no universally applicable formulation with Einstein's theory, incorporating the energy of gravitation itself.

(Penrose 1982, 52)

Implication of Equivalence?

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(But what does this rule out? Can gravitational energy depend on second derivatives of the metric, rather than just first derivatives? See Curiel 2017.)

Does T_{ab} include gravitational energy-momentum?

(See Curiel 2017, Dewar and Weatherall 2018; attributed to Geroch)

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Yes! ...

Consider spacetimes with $R_{ab} = 0$. Can represent gravitational waves (etc.), yet answer implies that gravitational EM vanishes.

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No! ...

Then the covariant conservation law $\nabla^a T_{ab} = 0$ applies to non-gravitational T_{ab}; gravitational EM sui generis, non-fungible.

(See Curiel 2017, Dewar and Weatherall 2018; attributed to Geroch)

Basic Challenge

No natural way of "adding up" energetic quantities at different points in a curved spacetime. (For example: mass-energy is one component of EM 4-vector, but generic spacetimes lack global notion of parallelism that would support adding mass-energy for extended body.)

Exceptions: Killing Symmetry

Consider a current $J_a = T_{ab}\xi^b$ with ξ^b unit timelike, then

$$\nabla^a J_a = \nabla^a (T_{ab}\xi^b) = (\nabla^a T_{ab})\xi^b + T_{ab}\nabla^{(a}\xi^{b)} \tag{5}$$

The first term vanishes ($\nabla^a T_{ab} = 0$).

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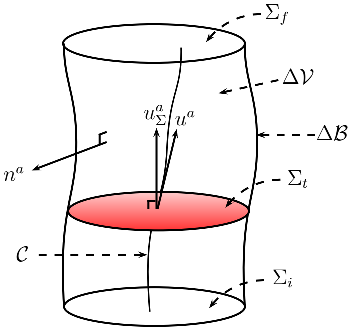
The first term vanishes ($\nabla^a T_{ab} = 0$).

If ξ^a is a Killing field, the second term also vanishes. Then J_a is a conserved current.

Integral Conservation Law



Observer moving along the curve $\mathcal{C}$ with velocity u_a.

Orthogonal vectors: n_a to the boundary, u^Σ_a to foliation Σ_t. Figure from Epp et al. (2013)

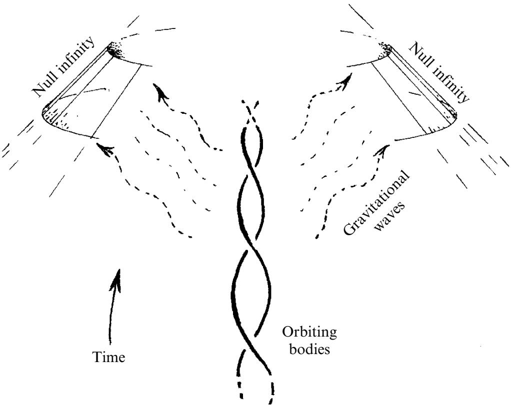
$$\int_{\Sigma_f - \Sigma_i} d\Sigma \;\; T_{ab} u^a_\Sigma \xi^b = \int_{\Delta B} dB \;\; T_{ab} n^a \xi^b - \int_{\Delta V} dV \;\; T_{ab} \nabla^{(a} \xi^{b)}$$

The second term vanishes if ξ^a is a Killing vector, and we recover the familiar form of an integral conservation law. (Then choosing ξ^a at a point fixes it throughout the volume.)

Exceptions: Asymptotic Symmetry

Asymptotic Infinities

- Null infinity: BMS group, positive mass theorems
- Spacelike infinity: ADM mass



(Image from Penrose 2004, 468)

It is an essential point of consistency, both in theory and observation, that the ripples of empty space that constitute the gravitational waves emitted by PSR $1913 + 16$ [Hulse-Taylor double neutron star] and other such systems indeed carry actual energy away. Gravitational energy is a genuinely non-local quantity. This does not imply that there is no mathematical description of gravitational energy, however. Although I believe it is fair to say that we do not yet have a complete understanding of gravitational mass/energy, there is an important class of situations in which a very complete answer can be given ... [Asymptotically flat, isolated systems.]

(Penrose 2004, 467)

Motivating Quasi-Local Definition

Gravitational energy for asymptotically flat systems: intrinsically global definition. No straightforward connection with intuitive picture of gravitational waves "carrying energy" through finite regions.

Eliminativist Position

> *There is no genuine energy-momentum conservation principle in GTR. [...] What typically hides this conclusion from view in these texts is the universal, almost desperate desire to make it seem as though there is such a principle at the heart of the theory.*
>
> *(Hoefer 2000, p. 195)*

(N.B: Hoefer focuses on pseudo-tensor approaches, and allows that quasi-local definitions may someday put "gravitational stress-energy onto firmer foundations.")

Current Philosophical Debate

- Eliminativists (Hoefer 2000; Duerr 2019a,b)

  Response to "substantivalist" argument: gravitational waves could knock down the rock of Gibraltar...

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- Responses
  - Background Dependent Structures (Lam 2010)
  - Foundations (Curiel 2017)
  - Functionalist (Read 2020)
  - Non-unique gravitational energy (Pitts 2010, ...)
  - Comparisons with Newtonian gravity (Dewar and Weatherall 2018, Duerr and Read 2019)
  - Noether's theorem; defense of pseudo-tensors and quasi-local energy (de Haro 2021)

Senses of Background Dependence

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- Definitions of non-local gravitational energy (pseudo-tensors, quasi-local) introduce background structure: not generally applicable, against the spirit of GR, usually rejected as "unphysical"...
- Main challenges:
  - Need analog of an "inertial frame" to define gravitational energy for extended bodies, ascriptions of energy-momentum relative to this choice
  - Lack of transformation properties to check consistency, and justification for use in particular applications
  - What are the invariant features of energy transmission via gravitational interactions?

Two Routes to Quasi-Local Quantities

Hamiltonian / Lagrangian Formulation

Brown and York (1993): given region S, τ_{ab} conjugate to the 3-metric induced on the boundary of S.

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Geometrical

Quantities for isolated systems defined in terms of: 2-surfaces "at infinity," introduce corresponding localized versions; or start with symmetric case and extend

(Szabados 2009, Lam 2010)

Pointilisme regarding Physical Properties

"[T]he doctrine that the history of the world is fully described by all the intrinsic properties of all the spacetime points and-or all the intrinsic properties at all the various times of point-sized bits of matter."

Philosophical anti-Pointilisme: Butterfield (2005)

Technical: how to treat invariance of physical properties defined over spacetime regions

Hawking Energy (1968)

Expression for energy $E_H(\mathcal{S})$ enclosed by $\mathcal{S}$ (roughly):

$$E_H(\mathcal{S}) = A + B \int_S \rho\rho' d\mathcal{S} \tag{6}$$

Measure of the "focusing" (ρ, ρ' convergence of incoming, outcoming null curves orthogonal to S) due to mass-energy, parameters A, B set by limiting behavior.

Drawbacks of $E_H(\mathcal{S})$

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Drawbacks of $E_H(\mathcal{S})$

- *Not* monotonic for family of surfaces $\mathcal{S}_r$, in general. Monotonicity proven for some cases: e.g., $\mathcal{S}_r$ foliate outgoing null hypersurfaces, matter sources satisfy dominant energy condition (Hawking 1968)
- Negative in Minkowski space for some choices of $\mathcal{S}$ (require convexivity condition on $\mathcal{S}$)

Nowadays, the state of the art is typically postmodern: Although there are several promising and useful suggestions, we have not only no ultimate, generally accepted expression for the energy-momentum ... but there is no consensus in the relativity community even on the general questions ... or on the list of criteria of reasonableness of such expressions.

(Szabados 2009, 9)

Natural Requirements for $E(\mathcal{S})$ (Szabados 2009)

- "Large sphere" behavior: recover results for isolated systems
- "Small sphere" behavior: behavior approaching a point, in vacuum and non-vacuum cases
- Other limiting behaviors: weak field limit, "round sphere" expression (for spherically symmetric spacetimes), marginally trapped surfaces
- Apply to arbitrary closed, orientable 2-surfaces
- Relationship to Hamiltonian / Lagrangian formalism

Conditions of Adequacy?

- Account of quasi-local energy applicable to all solutions?

(See also Jaramillo et al. 2010, Lam 2010)

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  ... or restrict to subspace of solutions (physically relevant, stability properties, etc. ...)

  ... for every element of chosen subspace there is an appropriate definition? (Exploiting background structure?)

(See also Jaramillo et al. 2010, Lam 2010)

Reference Frames and Conservation

- "Locally Asymptotic Reference Frames" in Tidal Heating
  (Thorne and Hartle 1985)
  Reference frame defined in buffer zone: source of external field far enough away for gravity to be weak, perturbative treatment

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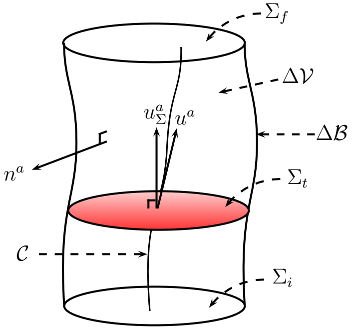
- "Locally Asymptotic Reference Frames" in Tidal Heating (Thorne and Hartle 1985)
  Reference frame defined in buffer zone: source of external field far enough away for gravity to be weak, perturbative treatment
- Proposal (McGrath, Epp, Mann) "Quasi-Local Rigid Frames": congruence of timelike worldlines, boundary of a finite spatial volume

Reference Frames and Quasi-Local Energy



Observer moving along the curve $\mathcal{C}$ with velocity u_a.

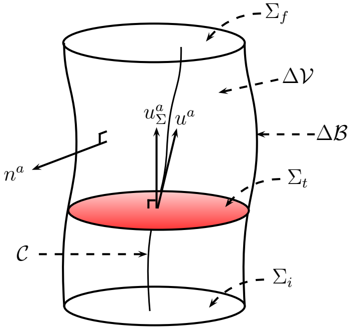
Orthogonal vectors: n_a to the boundary, u_a^Σ to foliation

Σ_t. Figure from Epp et al. (2013)

- Proposed quasi-local conservation law over region V
- Integration restricted to surface densities rather than over volumes
- Decompose Brown-York tensor τ_{ab} into components: energy, momentum, and stress (defined on the surface)

(see Epp, McGrath, Mann 2013; McGrath et al. 2012; Oltean et al. 2021)

Reference Frames and Quasi-Local Energy



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- Roughly: "bulk" term ($T_{ab}\nabla^{(a}\xi^{b)}$) breaks into "stress times strain" and "momentum times acceleration" terms
- "Quasi-local rigid frame": congruence of timelike curves on the boundary, with no expansion or shear, so that "stress times strain" vanishes

(see Epp, McGrath, Mann 2013; McGrath et al. 2012; Oltean et al. 2021)

Implications

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- *In the context of general relativity, the presence of momentum (matter or gravitational) flowing through the system causes the observers' local "radial" vector to precess relative to inertial gyroscopes (i.e., a frame dragging effect), and the vector cross product between this precession rate (the gravitational analogue of a magnetic field) and the observers' acceleration (the gravitational analogue of an electric field) corresponds to a flow of gravitational energy into the system.*

(McGrath et al., 2012, 13)

Eliminativism Revisited

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2. Background Dependence
   - Expect dependence on specification of reference frames to define energetic properties
   - Current status: ambiguous transformation properties, invariants

Tidal Heating Revisited

Invariance of Tidal Work

- Purdue (1999), Favata (2001): Ambiguity in localization of gravitational energy (exhibited by pseudo-tensorial calculations), ascribed to energy of the field or the matter source.
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- Booth and Creighton (2000): Treatment of tidal heating using Brown-York quasi-local energy, recovers same expression.
- All calculations yield same result for tidal work $\frac{dW}{dt}$, but no way to directly "transform" from one expression for gravitational energy to another